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ON SOME NEW γ-TYPE MAPS ON TOPOLOGICAL SPACES

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Abstract

In the present paper, we define and introduce a new type of topological transitive map called γ-type transitive and investigate some of its properties in (X, τ^{γ}), τ^{γ} denotes the γ-topology of a given topological space (X, τ). Relationships with some other type of transitive maps are given. We list some relevant properties of the γ-type transitive map. Further, we introduce the notions of γ-minimal mapping. We have proved that every topologically γ-type transitive map is a transitive map, but the converse not necessarily true, and that every γ-minimal map is a minimal map, but the converse not necessarily true.

1. Introduction

Maps and of course irresolute maps stand among the most important notions in the whole of pure and applied mathematical science. Various interesting problems arise when one considers openness. Its importance

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is significant in various areas of mathematics and related sciences. In 1972, Crossley and Hildebrand [17] introduced the notion of irresoluteness. Many different forms of irresolute maps have been introduced over the years. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [5] under the name of γ-open sets. The class of γ-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. The class of γ- open sets generates the same topology as the class of preopen sets. A subset A of a topological space X is said to be γ-*open* [5] (resp., α-*open* [6]), if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$ (resp., $A \subseteq Int(Cl(Int(A)))$. The complement of γ- open set is called γ- *closed*. The family of all γ- open (resp., α-open) subsets of (X, τ) will be denoted by τ^{γ} or $\gamma O(X)$ (resp., τ^{α} or $\alpha O(X)$). Throughout this paper, the word "space" will mean topological space. Recently, there has been some interest in the notion of a locally closed subset of a space. According to Bourbaki [16], a subset S of a space (X, τ) is called *locally closed*, if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [13] and [14] to define the concept of LC-continuity, i.e., a function $f : (X, \tau) \to (X, \sigma)$ is LC-continuous, if the inverse with respect to f of any open set in Y is closed in X. The study of semi open sets and semi continuity in topological spaces was initiated by Levine [6]. Bhattacharya and Lahiri [8] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets, which was introduced by Levine [5]. The collections of semi-open, semi-closed sets, and α-sets in (X, τ) will be denoted by $SO(X, \tau)$, $SC(X, \tau)$, and τ^{α}, respectively. Ogata [7] has shown that τ^{α} is a topology on X with the following properties: $\tau \subseteq \tau^{\alpha}$, $(\tau^{\alpha})^{\alpha} = \tau^{\alpha}$, and $S \in \tau^{\alpha}$ if and only if $S = U \setminus N$, where $U \in \tau$ and N is nowhere dense (i.e., $Int(Cl(N)) = \varphi$) in (X, τ). Hence $\tau = \tau^{\alpha}$ if and only if every nowhere dense (nwd) set in (X, τ) is closed. Clearly, every α-set is semi-open and every nwd set in (X, τ) is semi-closed. Andrijevic [1] has observed that $SO(X, \tau^{\alpha}) = SO(X, \tau)$, and that $N \subseteq X$ is nwd in $(X, \tau^{\alpha}) \Leftrightarrow N$ is nwd in (X, τ).

In this paper, we will define a new class of topological transitive maps called *topological* γ- *transitive* and a new class of γ- minimal maps. We will also study some of their properties. Relationships with some other type of transitive maps are given. We list some relevant properties of γ- type transitive map. Finally, one can see Murad Kaki [27] and [28] for some other new types of topological transitive maps.

2. Preliminaries and Definitions

Definition 2.1. By a topological system, we mean a pair (X, f), where X is a topological space (the phase space), and $f : X \to X$ is a continuous function. The dynamics of the system is given by $x_{n+1} = f(x_n)$, $x_0 \in X$, $n \in \mathbf{N}$ and the solution passing through x is the sequence $\{f(x_n)\}$, where $n \in \mathbf{N}$.

Definition 2.2. A continuous map $f : X \to X$ is called *topologically mixing*, if given any nonempty open subsets $U, V \subseteq X$ $\exists N \geq 1$, such that $f^n(U) \cap V \neq \phi$ for all $n > N$. Clearly, if f is topologically mixing, then it is also transitive but not conversely.

Devaney's definition of chaos. Let X be a metric space. A continuous map $f : X \to X$ is said to be chaotic on X, if

(1) f is transitive;

(2) the set of all periodic points of the map f is dense in X;

(3) f has sensitive dependence on initial conditions.

Theorem 2.3. *If* $f : X \to X$ *is transitive and has dense periodic points, then f has sensitive dependence on initial conditions.*

First of all, any definition of chaos must face the obvious question [1]: Is it preserved under topological conjugation ? That is to say, if f is chaotic and if we have a commutative diagram

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[d, "h"'] & X \arrow[d, "h"] \\ Y \arrow[r, "g"'] & Y \end{tikzcd}$$

where Y is another metric space and h is a homeomorphism, then is g necessarily chaotic?

Certainly, transitivity and the existence of dense periodic points are preserved as they are purely topological conditions. However, sensitivity is a metric property and in general, it is not preserved under topological conjugation.

Definition 2.4. The system (X, d, f), where X is infinite set (the phase space), d is a metric on X, and $f : X \to X$ is a continuous map (the evolution equation). [**2**] is Devaney chaotic if and only if for each $U \subseteq X, U \neq \phi$ and open, for each $V \subseteq X$ and $V \neq \phi$ and open, there exists periodic point $p \in U$, there exists $n \in \mathbf{N}$ such that $f^n(p) \in V$.

Definition 2.5. A topological space (X, τ) is irreducible, if every pair of nonempty open subsets of the space X has a nonempty intersection.

Theorem 2.6. *Prove that a topological space* (X, τ) *is an irreducible space if and only if each nonempty* γ- *open set W is dense in* (X, τ^{γ}).

Theorem 2.7. (X, τ) *is irreducible if and only if* (X, τ^{γ}) *is irreducible.*

Theorem 2.8. *Let* (X, τ) *be a topological space and let* $Y \subseteq X$. *Then the following conditions are equivalent*:

(1) *Y is dense in* (X, τ^{γ}).

(2) *An improper* γ *-closed subset containing Y is X.*

(3) $Int_{\gamma}(Y) = X$.

3. Applications of γ -Open Sets

Definition 3.1. Let A be a subset of a space X. Recall that a point $x \in X$ is said to be γ- *limit point* of A, if for each γ- open set U containing x, $U \cap (A \setminus \{x\}) \neq \phi$. The set of all γ-limit points of A is called a γ- *derived set of* A and is denoted by $D_{\gamma}(A)$.

A set U is dense in X if for any x in X either x in U or x is a limit point for U. Note that if A is closed and $U = X \setminus A$ is a dense set in X, then $Int(A) = \phi$.

Theorem 3.2. *For subsets* A *and* B *of a space* X, *the following statements hold*:

(1) $D_{\gamma}(A) \subseteq D(A)$, *where* $D(A)$ *is the derived set of* A;

(2) *if* $A \subseteq B$, *then* $D_{\gamma}(A) \subseteq D_{\gamma}(B)$;

(3) $D_{\gamma}(A) \cup D_{\gamma}(B) \subseteq D_{\gamma}(A \cup B)$ *and* $D_{\gamma}(A \cap B) \subseteq D_{\gamma}(A) \cap D_{\gamma}(B)$;

(4) $[D_{\gamma}(D_{\gamma}(A)) - A] \subseteq D_{\gamma}(A)$;

(5) $D_{\gamma}[A \cup D_{\gamma}(A)] \subseteq A \cup D_{\gamma}(A)$.

Example 3.3. Let $X = \{a, b, c\}$ with topology $\tau = \{\varphi, \{a\}, X\}$. Thus, we have $\tau^{\gamma} = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$.

Definition 3.4 ([30], p.178)**.** A point $x \in X$ is said to be a γ- *interior point of A*, if there exists a γ- open set U containing x such that $U \subseteq A$. The set of all γ- interior points of A is said to be γ- interior of A and denoted by $Int_\gamma(A)$. A is γ- open if and only if $Int_\gamma(A) = A$.

Definition 3.5. A subset A of a space X is said to be

(1) α- open [7] if $A \subseteq Int(Cl(Int(A)))$;

(2) semi-open [6] if $A \subseteq Cl(Int(A))$;

(3) pre-open [21] if $A \subseteq Int(Cl(A))$;

(4) β- open [22] if $A \subseteq Cl(Int(Cl(A)))$;

(5) γ- open [5] if $A \subseteq Int(Cl(A)) \cup Cl(Int(A))$.

Definition 3.6 ([2])**.** A space X is said to be γ- *compact*, if every γ- open cover of X has a finite subcover. A subset A of a space X is said to be γ- *compact relative to X*, if every cover of A by γ-open sets of X has a finite subcover.

Lemma 3.7 ([23])**.** *If U is an open set, then* $Cl(U \cap A) = Cl(U \cap Cl(A))$ *and hence* $U \cap Cl(A) \subseteq Cl(U \cap A)$.

Theorem 3.8. ([29] p.7)**.** *For subsets A and B of a space X, the following statements are true*:

(1) $Int_\gamma(A)$ *is the largest* γ- *open set contained in A*;

(2) *A is* γ- *open if and only if* $Int_\gamma(A) = A$;

(3) $Int_\gamma[Int_\gamma(A)] = Int_\gamma(A)$;

(4) $Int_\gamma(A) \cup Int_\gamma(B) \subseteq Int_\gamma(A \cup B)$;

(5) $Int_\gamma(A \cap B) \subseteq Int_\gamma(A) \cap Int_\gamma(B)$.

Example 3.9. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Thus,

$$\tau^{\gamma} = \{\phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}.$$

Definition 3.10. A topological space (X, τ) is said to be $\gamma - T_2$ [24] (resp., $\theta - T_2$ [26]), if for each pair of distinct points x and y of X, there exist disjoint γ-open (resp., θ-open) subsets U and V of X containing x and y, respectively.

Definition 3.11. A topological space (X, τ) is said to be θ-compact [26] (resp., γ- *compact* [25]), if every θ-open (resp., γ- open) cover of X has a finite subcover. A subset A of a topological space X is said to be θ-compact relative to X, if every cover of A by θ-open subsets of X has a finite subcover of A.

Definition 3.12 ([5])**.** A space X is called γ- *regular*, if for each γ- closed set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.

Definition 3.13 ([5]**).** A space X is said to be γ- *normal*, if for every pair of disjoint γ- closed subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

4. γ -Type Transitive Maps

In this section, we generalize topologically transitive maps to γ- type transitive maps that implies transitive maps, but the convers not necessarily true. We define the γ -type transitive maps on a space (X, τ) and γ- type minimal maps that implies minimal but not conversely, and we study some of their properties and prove some results associated with these new definitions. We investigate some properties and characterizations of such maps.

Definition 4.1. Recall that a subset A of a space X is called γ- *dense* in X if $Cl_\gamma(A) = X$, we can define equivalent definition that a subset A is said to be γ- dense, if for any x in X either x in A or it is a γ -limit point for A.

Remark 4.2. Any γ -dense subset in X intersects any γ- open set in X.

Proof. Let A be a γ- dense subset in X, then by definition, $Cl_\gamma(A) = X$, and let $U \neq \phi$ be a γ- open set in X. Suppose that $A \cap U = \phi$. Therefore, $B = U^c$ is γ- closed because B is the complement of γ- open and $A \subset U^c = B$. So $Cl_\gamma(A) \subset Cl_\gamma(B)$, i.e., $Cl_\gamma(A) \subset B$, but $Cl_\gamma(A) = X$, so $X \subset B$, this contradicts that $U \neq \phi$.

Definition 4.3. A function $f : X \to X$ is called γ -*irresolute*, if the inverse image of each γ- open set is a γ- open set in X.

Example 4.4. Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a, b\}\}$. Then $\gamma O(X, \tau) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and the set of all γ- closed sets is $\gamma C(X, \tau) = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}, \{b\}, \{a\}\}$. Then define the map $f : X \to X$ as follows: $f(b) = a, f(c) = c, f(a) = b,$ we have f is γ- irresolute because $\{a\}$ is γ- open and $f^1(\{a\}) = \{b\}$ is γ- open; and $\{b\}$ is γ- open and $f^1(\{b\}) = \{a\}$ is γ- open and $f^{-1}(\{a, b\}) = \{a, b\}$ is γ- open; $\{b, c\}$ is γ- open and $f^1(\{b, c\}) = \{a, c\}$ is γ- open; so f is γ- irresolute.

Definition 4.5. A subset A of a topological space (X, τ) is said to be *nowhere* γ- *dense*, if its γ- closure has an empty γ- interior, that is, $int_\gamma(Cl_\gamma(A)) = \phi$.

We first introduce the definitions of topological γ-type transitive maps and γ- dense orbit. Let (X, τ) be a topological space, $f : X \to X$ be γ- irresolute map.

Definition 4.6. f is called γ*-type transitive*, if every pair of non-empty γ- open sets U and V in X, there is a positive integer n such that $f^n(U) \cap V \neq \phi$.

Definition 4.7. If for $x \in X$, the set $\{f^n(x): n \in \mathbf{N}\}$ is dense in X, then x is said to have a dense orbit. If there exists such an $x \in X$, then f is said to have a dense orbit.

Definition 4.8. A function $f : X \to X$ is called γr*-homeomorphism* if f is γ-irresolute bijective and $f^{-1} : X \to X$ is γ-irresolute.

Definition 4.9. Two topological systems $f : X \to X$, $x_{n+1} = f(x_n)$ and $g : Y \to Y$, $y_{n+1} = g(y_n)$ are said to be topologically γr-conjugate, if there is γr-homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$ (i.e., $h(f(x)) = g(h(x))$). We will call h a topological γr-conjugacy.

Remark 4.10. If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$, then $\{y_0 = h(x_0), y_1 = h(x_1), y_2 = h(x_2), ..\}$ yields an orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$. In particular, h maps periodic orbits of f onto periodic orbits of g.

We introduced and defined the new type of transitive in such a way that it is preserved under topologically γr-conjugation. We can state the following proposition:

Proposition 4.11. *If* $f : X \to X$ *and* $g : Y \to Y$ *are topologically* γr*- conjugate. Then*

(1) *f is topologically* γ*- transitive if and only if g is topologically* γ*- transitive*;

(2) *f is* γ- *minimal if and only if g is* γ- *minimal*;

(3) *f is topologically* γ- *mixing if and only if g is topologically* γ- *mixing*.

Associated with the new Definition 4.6, we can prove the following new theorem:

Theorem 4.12. *For a* γ- *irresolute map* $f : X \to X$, *where X is a topological space*, *the following are equivalent*:

(1) *f is topologically* γ- *type transitive*;

(2) *any proper* γ- *closed subset* $A \subset X \ni f(A) \subseteq A$ *is nowhere* γ- *dense*;

(3) $\forall A \subseteq X \ni f(A) \subseteq A$, A *is either* γ *-dense or nowhere* γ *-dense*;

(4) *any subset* $A \subseteq X \ni f^{-1}(A) \subseteq A$ *with non-empty* γ *-interior is* γ- *dense*.

Proof.

$(1) \Rightarrow (2)$: If f is topologically γ-type transitive and $A \subseteq X$ is γ- closed and $f(A) \subseteq A$, i.e., A is invariant), then $V = X \setminus A$ is non-empty and γ- open since it is the compliment of γ -closed. We cannot have $U = Int_\gamma(A) \neq \phi$, since f is γ- type transitive, so there exists some $n \geq 1$ such that $f^n(U) \cap V \neq \phi$, contradicting the invariance of A. This shows A is nowhere γ- dense.

$(2) \Rightarrow (3)$: Let $A \subseteq X$ be invariant (i.e., $f(A) \subseteq A$). Then, since f is γ- irresolute $f(Cl_\gamma(A)) \subseteq Cl_\gamma(f(A)) \subseteq Cl_\gamma(A)$. This shows $Cl_\gamma(A)$ is invariant, so either $Cl_\gamma(A) = X$ or A is nowhere γ- dense by (2). Thus either A is γ- dense or nowhere γ- dense.

$(3) \Rightarrow (4)$: Let $A \subset X$ and $f^{-1}(A) \subset A$ with non-empty γ- interior. Then $X \setminus A$ is invariant and hence either γ- dense or nowhere γ- dense by (3). But $X \setminus A$ γ -dense gives $Int_{\gamma}(A) = \phi$ contrary to hypothesis, so $X \setminus A$ must be nowhere γ- dense. This gives $Int_{\gamma}(X \setminus A) = \phi$, so A is γ- dense.

$(4) \Rightarrow (2)$: Let $A \subset X$ be γ- closed and invariant. Then $U = X \setminus A$ is non-empty and γ- open, so U is γ- dense by (4), so we have $Int_{\gamma}(A) = \phi$. Thus A is nowhere γ- dense.

However, conditions (2), (3), and (4) do not implies condition (1), as an example we give a function, which has a γ- dense orbit but which is not topologically γ- type transitive as follows.

Example 4.13. Let $X = \{x, y\}$ with $\tau^{\gamma} = \{\phi, \{x\}, \{y\}, X\}$ and let $f : X \to X$ be defined as the constant function $f(x) = f(y) = y$.

Namely, the orbit of x is γ- dense. But f is not topologically γ- type transitive: If we choose $U = \{y\}$ and $V = \{x\}$, then there is no n with $f^{n}(U) \cap V \neq \phi$.

5. Minimal Systems

By a topological system (X, f), we mean a topological space X together with a continuous map $f : X \to X$. The space X is sometimes called the phase space of the system (X, f). A subset X of A is called f *-invariant* if $f(A) \subseteq A$.

Definition 5.1. A system (X, f) is called *minimal* if X does not contain any non-empty, proper, closed *f-invariant* subset. In such a case, we also say that the map f itself is minimal.

Given a point x in a system (X, f), $O_f(x) = \{x, f(x), f^2(x), \dots\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its ω-limit set, i.e., the set of limit points of the sequence $x, f(x), f^2(x), \dots$.

The following conditions are equivalent:

(1) (X, f) is minimal.

(2) Every orbit is dense in X.

(3) $\omega_f(x) = X$ for every $x \in X$.

A minimal map f is necessarily surjective if X is assumed to be Hausdorff and compact. Now, we will study the existence of minimal sets. Given a dynamical system (X, f), a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed, and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f). The basic fact discovered by Birkhoff is that in any compact system (X, f), there are minimal sets. This follows immediately from the Zorn's lemma. Since any orbit closure is invariant, we get that *any compact orbit closure contains a minimal set*. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) either are disjoint or coincide. A minimal set A is strongly *f-invariant*, i.e., $f(A)$ is a subset of A, provided it is compact Hausdorff.

A topological system (X, f) is called γ- *minimal*, if X does not contain any non-empty, proper, γ- closed *f*-invariant subset. In such a case, we also say that the map f itself is γ- minimal. Let us begin with an equivalent definition.

Definition 5.2 (γ-minimal)**.** Let X be a topological space and f be γ- continuous function on X. Then (X, f) is called γ- minimal system (or f is called γ- minimal function on X), if one of the three equivalent conditions hold:

(1) The orbit of each point of X is γ- dense in X.

(2) $Cl_\gamma(O_f(x)) = X$ for each $x \in X$.

(3) Given $x \in X$ and a nonempty γ- open $U \subset X$, there exists $n \in \mathbf{N}$ such that $f^n(x) \in U$.

Theorem 5.3. *For topological system* (X, f), *the following statements are equivalent*:

(1) *f is a* γ- *minimal function.*

(2) *If* E *is a* γ- *closed subset of* X *with* $f(E) \subset E$, *we say* E *is invariant. Then* $E = \phi$ *or* $E = X$.

(3) *If U is a nonempty* γ *-open subset of X, then* $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

Proof.

(1) $\Rightarrow$ (2): If $A \neq \phi$, let $x \in A$. Since A is invariant and γ -closed, i.e., $Cl_\gamma(A) = A$, so $Cl_\gamma(O_f(x)) \subset A$. On the other hand, $Cl_\gamma(O_f(x)) = X$. Therefore $A = X$.

(2) $\Rightarrow$ (3): Let $A = X \setminus \bigcup_{n=0}^{\infty} f^{-n}(U)$. Since U is nonempty, $A \neq X$ and since U is γ-open and f is γ- irresolute, A is γ -closed. Also $f(A) \subset A$, so A must be an empty set.

(3) $\Rightarrow$ (1): Let $x \in X$ and U be a nonempty γ-open subset of X. Since $x \in X = \bigcup_{n=0}^{\infty} f^{-n}(U)$. Therefore $x \in f^{-n}(U)$ for some $n > 0$. So $f^n(x) \in U$.

Proposition 5.4. *Let X be a γ-compact space without isolated point, if there exists a γ-dense orbit, that is there exists $x_0 \in X$ such that the set $O_f(x_0)$ is γ-dense, then the function f is γ-type transitive.*

Proof. Let $x_0 \in X$ be such that $O_f(x_0)$ is γ-dense in X. Given any pair U, V of γ-open subsets of X, by γ-density, there exists n such that $f^n(x_0) \in U$, but $O_f(x_0)$ is γ-dense implies that $O_f(f^n(x_0))$ is γ-dense, so by Remark 4.2 intersects V, i.e., there exists m such that $f^m(f^n(x_0)) \in V$. Therefore $f^{m+n}(x_0) \in f^m(U) \cap V$. That is, $f^m(U) \cap V \neq \varphi$. So f is γ-type transitive.

6. Conclusion

There are the main results of this paper:

Proposition 6.1. *Every topologically γ-transitive map is a topologically transitive map as every open set is γ-open set, but the converse not necessarily true.*

Proposition 6.2. *Every γ-minimal map is a minimal map as every open set is γ-open set, but the converse not necessarily true.*

Theorem 6.3. *For an γ-irresolute map $f : X \to X$, where X is a topological space, the following are equivalent:*

(1) *f is topologically γ-type transitive;*

(2) *any proper γ-closed subset $A \subset X \ni f(A) \subseteq A$ is nowhere γ-dense;*

(3) $\forall\ A \subseteq X \ni f(A) \subseteq A$, *A is either γ-dense or nowhere γ-dense;*

(4) *any subset* $A \subseteq X \ni f^{-1}(A) \subseteq A$ *with non-empty* γ- *interior is* γ- *dense*.

Theorem 6.4. *For a topological system* (X, f), *the following statements are equivalent*:

(1) *f is a* γ-*minimal function*.

(2) *If* E *is a* γ- *closed subset of* X *with* $f(E) \subset E$, *we say* E *is invariant*. *Then* $E = \phi$ *or* $E = X$.

(3) *If U is a nonempty* γ- *open subset of X*, *then* $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

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